QUEEN'S UNIVERSITY
APSC 171J – Quiz #2: Solution
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Written: February 13, 2013

INSTRUCTIONS

- This quiz is being written in the tutorial (9:30-10:20am) Wednesday, February 13
- Answer all questions, writing clearly on the sheets provided.
- One mark in each question is for a **fully** correct solution, which **must** be placed in the box provided
- Whenever possible, simplify your solution.
- There are no part marks: you will receive only integer marks for each question.

FOR INSTRUCTOR'S USE ONLY		
Question	Mark Available	Received
1	6	
2	10	
TOTAL	16	

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1. [6 marks] Find the definite integral

$$\int_1^2 t^2 \ln(2t)dt.$$

(**[5 marks]** for process, **[1 mark]** for final answer in box below)

Use the method of Integration by Parts, and choose the $\ln$ function as having the "easier" derivative. Thus: Thus,

$$\begin{aligned} u &= \ln(2t) & v &= \tfrac{1}{3}t^3 \\ du &= \tfrac{1}{2t}\cdot 2dt & dv &= t^2 dt \end{aligned}$$

$$\begin{aligned}
\int_1^2 t^2 \ln(2t)dt &= \frac{1}{3}t^3 \ln(2t)\Big|_1^2 - \int_1^2 \frac{1}{3}t^3\frac{1}{2t}2dt \\
&= \frac{1}{3}t^3 \ln(2t)\Big|_1^2 - \int_1^2 \frac{1}{3}t^2 dt \\
&= \frac{1}{3}t^3 \ln(2t)\Big|_1^2 - \frac{1}{3}\left[\frac{1}{3}t^3\right]_{t=1}^2 \\
&= \frac{1}{3}(2)^3\ln(4) - \frac{1}{3}(1)^3\ln(2) - \frac{1}{3}\left(\frac{1}{3}(2)^3\right) + \frac{1}{3}\left(\frac{1}{3}(1)^3\right) \\
&= \frac{8}{3}\ln(4) - \frac{1}{3}\ln(2) - \frac{8}{9} + \frac{1}{9} \\
&= 2.688.
\end{aligned}$$

Final Answer:

$$\int_1^2 t^2 \ln(2t)dt = 2.688.$$

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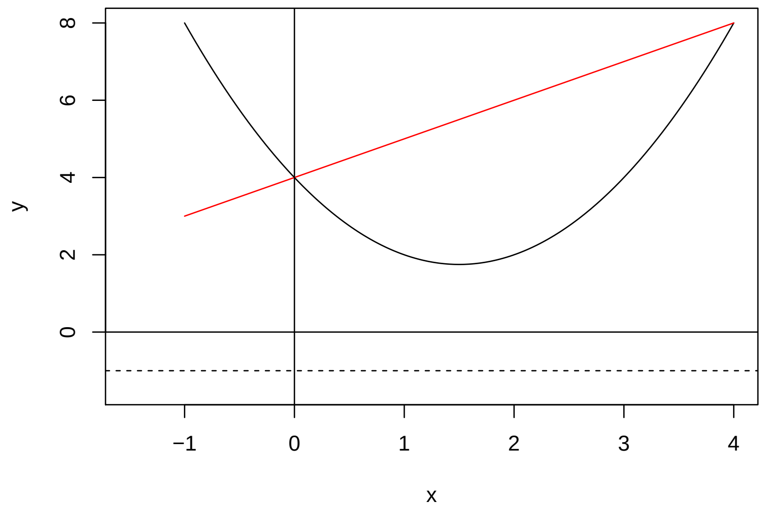
2. [10 marks] Find the volume generated by rotating about the line $y = -1$ the region bounded by the graphs of the equations $y = x^2 - 3x + 4$ and $y = 4 + x$. Include a clear, large, **labeled** diagram, and state clearly which approach you are using (slices vs. shells).

(**[2 marks]** for diagram, **[3 marks]** for correct volume integral statement, **[4 marks]** for solving the integral properly, and **[1 mark]** for final numeric volume in box on the next page)

Start by drawing a clear diagram:



and then decide whether you are using Shells or Slices. From the diagram, as the area does not pass the horizontal line test (which is parallel), it will be easier to use Slices rather than Shells. Thus, we need inner and outer radii:

$$r_{inn} = \left(x^2 - 3x + 4\right) + 1$$
$$R_{out} = (4 + x) + 1$$

and then set up the volume integral. We need to know the limits on our slices, which run from $x = 0$ to $x = 4$, so

$$\begin{aligned}
V &= \pi \int_0^4 \left[(4 + x + 1)^2 - \left(x^2 - 3x + 4 + 1\right)^2\right] dx \\
&= \pi \int_0^4 \left[25 + 10x + x^2 - \left(x^4 - 3x^3 + 5x^2 - 3x^3 + 9x^2 - 15x + 5x^2 - 15x + 25\right)\right] dx \\
&= \pi \int_0^4 \left[25 + 10x + x^2 - \left(x^4 - 6x^3 + 19x^2 - 30x + 25\right)\right] dx \\
&= \pi \int_0^4 \left[25 + 10x + x^2 - x^4 + 6x^3 - 19x^2 + 30x - 25\right] dx \\
&= \pi \int_0^4 \left[40x - 18x^2 - x^4 + 6x^3\right] dx
\end{aligned}$$

extra space for Question 2

$$
\begin{aligned}
V &= \pi \int_0^4 \left[40x - 18x^2 - x^4 + 6x^3\right] dx \\
&= \pi \left[20x^2 - 6x^3 - \frac{1}{5}x^5 + \frac{6}{4}x^4\right]_0^4 \\
&= \pi \left[\left(20(4)^2 - 6(4)^3 - \frac{1}{5}(4)^5 + \frac{6}{4}(4)^4\right) - \left(20(0)^4 - 6(0)^3 - \frac{1}{5}(0)^5 + \frac{6}{4}(0)^4\right)\right] \\
&= 115.2\pi = \frac{576}{5}\pi.
\end{aligned}
$$

Final Answer:

The volume of this solid is $\frac{576}{5}\pi \approx 361.91$ cubic units.