Student Number _____

Instructor _____ Section _____

QUEEN'S UNIVERSITY, FACULTY OF APPLIED SCIENCE
APSC 171 FINAL EXAMINATION, DECEMBER, 2010
L. JONKER, M. ROTH, N. YUI

HAND IN
answers recorded
on question paper

- The candidate is urged to submit with the answer paper a clear statement of any assumptions made if doubt exists as to the interpretation of any question that requires a written answer.
- PLEASE NOTE: Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.
- Answer in the spaces provided on the question paper. If necessary, an answer may be continued on THE BACK OF THE PREVIOUS PAGE.
- You may use calculators with a GOLD sticker.
- SHOW HOW YOU REACH YOUR RESULTS. Marks are not given for a correct answer alone. State or display answers in an appropriate way.
- You may write in pencil, but write clearly. Do not write in red ink.
- Except where a decimal answer is asked for, it is preferable to leave answers in the form $\sqrt{\pi}$, e^2 and so on. However, do any obvious simplification (for example $2 + \frac{1}{2} + \frac{1}{3} = 2\frac{5}{6}$ or $\frac{17}{6}$, $\frac{(x+1)^2}{(x+1)} = (x+1)$).
- Marks per question or part question are shown in square brackets on the right margin (for example [4]). The total number of marks is 60.
- Check that your question paper has 10 pages.

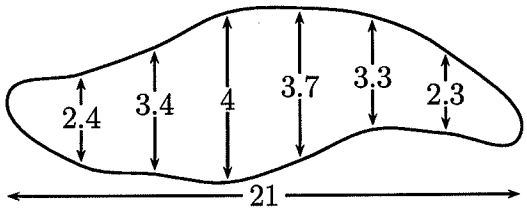
FOR EXAMINER'S USE ONLY		
Page	Mark Available	Mark
2	11	
3	7	
4	6	
5	6	
6	8	
7	8	
8	2	
9	5	
10	7	
Total	60	

1. Find the derivative dy/dx when $y = (\arctan(3x))^{x+1}$ **[6]**

2. Calculate the integral $\int_1^\infty \frac{x+\sqrt{x}}{x^{5/2}}\,dx$ **[5]**

3. Calculate the limit $\lim_{x \to 0} \dfrac{\cos 5x - \cos 7x}{x^2}$. **[3]**

4. The widths (in meters) of a pond were measured at 3-meter intervals as indicated in the figure. Use the trapezoidal rule to estimate the area of the surface of the pond. **[4]**



5. Calculate the integral $\int_0^1 \frac{3x-1}{x^2+x-6}\,dx$ **[6]**

6. Use integration by parts to calculate the indefinite integral

$$\int \frac{x^{11}}{(x^6+1)^{5/2}}\,dx\,.$$

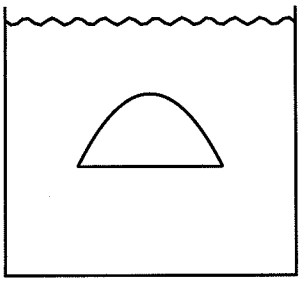
[6]

7. The Great Pyramid of Giza is thought to have been 146 meters high on a square base whose sides measured 230 meters when it was completed around 2560 BC. It was made of limestone, whose density is $\rho = 2,360\ \text{kg/m}^3$. The interior is solid throughout except for some hollow chambers, which will be ignored for the purpose of this problem. Use an integral to calculate the total amount of work done in getting the limestone blocks off the desert surface into place on the structure. [8]

8. Find the volume of the solid generated by rotating the region bounded by $x = (y-3)^2$ and the lines $x = 4$, about the line $x = -1$. [8]

9. When a container holds a fluid, the fluid exerts pressure on the walls (and bottom) of the container. Pressure is force (exerted by the fluid on the container wall) per unit area and is measured in N/m^2 (or "Pascals"). It is known that the magnitude of this pressure varies with distance h below the fluid surface according to the formula $p = \rho gh$, where p is the pressure, ρ is the mass density of the fluid in kg/m^2, h is depth below the fluid surface (in meters), and g is acceleration due to gravity (9.8 m/s^2). A fish tank at a public aquarium is to have a glass window whose top edge is in the shape of a parabola and whose bottom edge is horizontal. The window is to be one meter high and two meters wide at the base, and it will be set in the side wall of the aquarium so that the bottom of the window is two meters below the surface of the water in the tank and 1.5 meters above the bottom of the tank. It is important to calculate the total *force* on this window in order to determine the required thickness of glass and to design the frame that will hold it in place. Assume that the mass density of water is 1000 kg/m^3.

   (a) Explain why it is not possible to solve this problem by simply multiplying the area of the window by the water pressure. [1]



   (b) Explain why the analysis of this problem should involve thin horizontal strips rather than thin vertical strips of the glass. [1]

Question 10 continued:

(c) Find an expression for the total force on a thin horizontal strip across the window, y meters below the top of the window, and use that to find an expression for the integral that equals the total force of the water on the window (you do not have to calculate the integral). Note that there is just one parabola that will fit the dimensions given for the window in the problem. **[5]**

10. A 1000 liter tank is full of brine (salt water). The total mass of salt in the tank is 15 kg initially. Less salty water, containing 1 kg of salt per 200 liters, is pumped into the tank at a rate of 10 liters per minute. The water in the tank is kept thoroughly mixed, and excess water escapes from a drain at the top of the tank. It can be shown that under these circumstances the amount of salt, $S(t)$, measured in kg, satisfies the differential equation

$$S'(t) = \frac{1}{20} - \frac{1}{100}S(t)\,.$$

How much salt will there be in the tank after 3 minutes? **[7]**